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中心的 p -群拡大である有限群のブロックについて

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This is a joint work with Naoko Kunugi. A result stated here will be published with a complete proof, see [1]. The result in [1] is, actually, inspired by a result stated in their paper [3] of Usami and Nakabayashi, where they prove our theorem for principal block algebras.

Here, we consider the following setting-up.

First of all, let G and G' be finite groups which have a common central p -subgroup Z for a prime number p , and let \overline{A} and \overline{A}' respectively be p -blocks of G/Z and G'/Z induced by p -blocks A and A' respectively of G and G' , both of which have the same defect group. Let (\mathcal{O}, K, k) be a splitting p -modular system for all subgroups of G and G' , that is, \mathcal{O} is a complete discrete valuation ring of rank one with its quotient field K of characteristic zero and with its residue field k of characteristic p , and both K and k are splitting fields for all subgroups of G and G' .

Then, we may have the following natural question. Namely,

Question. If \overline{A} and \overline{A}' are of a certain equivalence, then so are A and A' ?

Our main result is in fact the following.

Theorem (Koshitani-Kunugi). *Keep the notation above. Assume that G and G' have a common subgroup H satisfying $H \supseteq P \supseteq Z$ for a p -subgroup P of H and a central p -subgroup Z of G and G' . Let A and A' , respectively, be block algebras of $\mathcal{O}G$ and $\mathcal{O}G'$ such that P is a defect group of A and A' . Set $\overline{G} = G/Z$, $\overline{G}' = G'/Z$, $\overline{P} = P/Z$ and $\overline{H} = H/Z$, and let $\pi : \mathcal{O}G \twoheadrightarrow \mathcal{O}\overline{G}$ and $\pi' : \mathcal{O}G' \twoheadrightarrow \mathcal{O}\overline{G}'$ be the canonical \mathcal{O} -algebra-epimorphisms induced by the canonical group-epimorphisms $G \twoheadrightarrow \overline{G}$ and*

$G' \twoheadrightarrow \overline{G'}$, respectively. Write $\overline{A} = \pi(A)$ and $\overline{A'} = \pi'(A')$. Then, it is well-known that \overline{A} and $\overline{A'}$, respectively, are again block algebras of $\mathcal{O}\overline{G}$ and $\mathcal{O}\overline{G'}$ such that \overline{P} is a defect group of \overline{A} and $\overline{A'}$.

If there is an $(\overline{A}, \overline{A'})$ -bimodule \overline{M} such that $\overline{A} \otimes_{\mathcal{O}\overline{H}} \overline{A'} = \overline{M} \oplus (\text{projective})$ and \overline{M} realizes a Morita equivalence between \overline{A} and $\overline{A'}$, then A and A' are also Morita equivalent via an (A, A') -bimodule M such that $M|A \otimes_{\mathcal{O}H} A'$.

Remark. Theorem above is, actually, pretty much usable to prove Broué's abelian defect group conjecture for non-principal block algebras. For instance, see [2].

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References

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